

Problem Solving and Mathematical Argument

A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosity and brings into play your inventive facilities, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery.

George Polya
How to Solve It

One theme that will run throughout Math 300 will be to gain experience with problem solving, reasoning and proof. A portion of each week will be devoted to considering challenging problems. At first we might engage in experimentation, i.e. we look at various special cases, we work with physical models, etc. If we are fortunate, our experimentation will lead us to a conjecture, i.e. a conclusion reached by guessing or an opinion admittedly without sufficient evidence for proof. Once we have a conjecture, we will make the effort to solve the problem or provide a reasonable proof. The next step is communicating your solution, orally or in writing. Developing the ability to explain mathematics is part of gaining the ability to add value to the school experience of your students. Discussing mathematical ideas, reasons, goals, and relationships with your students will help them make sense of the mathematics they are learning.

Here is George Polya's four-step Problem-solving Process:

1. Understand the problem
Can you state the problem in your own words? What are you trying to find or do? What information, if any, is missing or not needed?
2. Devise a plan
Look for a pattern. Examine a simpler or special case of the problem. Make a table. Make a diagram. Write an equation. Use guess and check. Use indirect reasoning.
3. Carry out the plan
Implement the strategies from step 2. Keep an accurate record of your work.
4. Look back
Interpret your solution in terms of the original problem. Does your answer make sense? Does it answer the question asked? Is there another method of finding the solution? Is there a related or more general problem that you should consider?

To have a *complete* solution to a problem, you need first to investigate the problem until you think you have an answer to the question asked. Then you need to ask yourself, "How much do I need to say in order to show that my answer is correct?" It would be acceptable to substitute "prove" or "demonstrate" or some similar word for "show" that communicates the need to explain why the answer is correct.

One answer is: "When your explanation is sufficient for your audience both to understand that your answer is correct and to understand why your answer is correct." In particular, your explanation must be valid. How much one should say may depend upon what the audience will understand, but the person who has solved a problem should always be prepared to provide more detail if a listener doesn't understand the argument she (he) has offered. Thus, the goal here is to be able to communicate in a way that your audience (perhaps with some effort) understands the explanation.